~~Paper to be~~ Presented at the AIChE Natl
~~National~~ Meeting, Houston, ~~Texas~~
Dec. 1-5, 1963 ~~and Publication~~
~~in the AIChE Journal~~

N65-88880

~~X63 16118~~

Code 2A

29p

E-2155

DETERMINATION OF COMBINED CONDUCTION AND RADIATION OF HEAT THROUGH ABSORBING MEDIA BY THE EXCHANGE-FACTOR APPROXIMATION

By John R. Howell [1963] 10 p refs

Submitted for Publication

Lewis Research Center
National Aeronautics and Space Administration,
Cleveland, Ohio

ABSTRACT

(NASA TM X-50808) 16118

A simple method of solution for a group of problems involving the combined modes of radiation and conduction heat exchange through radiant media is proposed. Considering the radiant exchange factors involved in the equations of transfer to be independent of the conduction process replaces the integrals in these equations by constants that can be determined either from pure radiation solutions available in the literature or by an independent solution. Comparison of approximate radiation-conduction solutions is made with exact solutions of the integro-differential equations present in the literature and agreement is found over a large range of parameters.

Author

INTRODUCTION

Problems involving combined modes of heat transfer lead to the consideration of nonlinear integro-differential equations, the solution of which is of course quite difficult. For even simple cases involving conduction and radiation of heat between surfaces, few investigations have been carried out.

The problem considered here is the determination of the temperature distribution in a gray nonisothermal gas with constant thermal conductivity and absorption coefficient contained between infinite gray surfaces. Specifically, the cases of infinite gray parallel plates and infinitely long concentric gray cylinders are attacked. The heat transferred between the bounding surfaces is also considered.

The problem of infinite plates described in the preceding paragraph has been examined by some authors (1 to 3). The solutions of Viskanta and Grosh (1 and 3) are exact and are formulated in terms of integro-differential equations. Einstein (2) attacks the problem through consideration of zone interchange factors, which also leads to integro-differential equations. He treats finite plates and includes the effect of flow, but is forced to make certain approximations. Both the methods cited are tedious from the standpoint of solution of a specific problem, and a reduction in the form of the equations involved would be desirable.

In this paper, it is shown how problems of energy transfer by combined conduction and radiation in some simple geometries can be formulated in terms of differential equations, which can then be solved by a finite-difference technique. This removal of the integral terms involves an approximation of the radiative exchange factors used in the formulation. With this approximation, it is shown that, if the optical thickness, radiation-conduction parameter, surface-temperature ratio, surface emissivities, and, for concentric cylinders, the diameter ratio are known, then the temperature distribution in the gas and the heat transfer between the surfaces can be readily found. The solutions converge quite rapidly because of the absence of integral terms in the equations.

ANALYSIS

Case of Infinite Parallel Plates

Consider the geometry of infinite parallel gray plates at temperatures T_0 and T_1, separated by a distance D, and enclosing a gray gas of absorption coefficient κ and thermal conductivity k. The gas is nonisothermal, although the gas properties are considered independent of temperature. The problem is to find the temperature distribution in the gas and the heat transferred between the plates.

A heat balance for a unit time on an isothermal gas increment of width Δx, refractive index 1, and unit area gives

$$\epsilon_0 \sigma T_0^4 F_{0-\Delta x,i} + \epsilon_1 \sigma T_1^4 F_{1-\Delta x,i} + \frac{k}{\Delta x}(T_{g,i-1} - T_{g,i}) + \sum_{gi}$$

$$= \frac{k}{\Delta x}(T_{g,i} - T_{g,i+1}) + 4\kappa\sigma T_{g,i}^4 \Delta x \qquad (1)$$

The fraction $F_{A\,\Delta x,i}$ appearing in the first two terms is defined as the fraction of the energy emitted by surface A, which is absorbed in a gas increment i of width Δx at some time during transit to absorption at either surface.

The first two terms in Equation (1) represent, respectively, the rate of energy radiated from surfaces 0 and 1 and absorbed in increment i. These terms include that energy originally leaving a surface which is absorbed elsewhere in the gas and is then reemitted, finally being absorbed in increment i.

The third term represents the rate of energy conducted into increment i from the adjacent $(i - 1)$ increment.

The term $\sum_{gi}$ is included to account for energy that originally leaves a surface by conduction, is conducted into elements other than i, is then emitted as radiant energy by these elements, and is absorbed in increment i. It will consist of contributions from elements where the slope of temperature is decreasing (d^2T/dx^2 positive), since only in these elements will more energy be conducted in than out. This excess conduction acts as a source of radiant energy in the gas, and some portion of this radiant energy may be absorbed in increment i.

The first term on the right is the rate of energy conducted from increment i to adjacent increment $(i + 1)$. The final term is the radiation emitted from increment i. Because $\sum_{gi}$ will, in general, have contributions from only a portion of the elements and the total value of $\sum_{gi}$ will usually be small compared to the other energy entering element i, it will henceforth be neglected. If a somewhat more accurate final temperature distribution were desired, $\sum_{gi}$ could be carried forward in the following steps: it could be set equal to zero as a first approximation and then solved for by iteration on the basis of the resulting temperature distribution. In view of other approximations to be made, this accuracy was not deemed necessary.

The fraction $F_{A\,\Delta x,i}$ is dependent upon the conduction process, since, for a pure radiation problem, all energy absorbed in an increment will be reemitted under steady-state conditions, while, with simultaneous conduction, some portion of the absorbed energy will be conducted from any increment where

d^2T/dx^2 is negative. Thus, when conduction is present, the fraction of the radiation leaving a surface and finally reaching a given element will be smaller than in the absence of conduction.

When radiation is the prominent mode of heat transfer, however, any error that would occur from the assumption that $F_{A\ \Delta x,i}$ is independent of the conduction process is small. When conduction predominates, though, large errors in the radiation contribution may be tolerated. In the following argument, therefore, it is assumed that $F_{A\ \Delta x,i}$ is independent of the conduction process. This leads to substantial reduction in the complexity of the equations of transfer.

If surfaces 0 and 1 are at the same temperature, Equation (1) reduces to

$$\epsilon_0 F_{0-\Delta x,i} + \epsilon_1 F_{1-\Delta x,i} = 4\kappa\ \Delta x \tag{2}$$

Since this relation is independent of surface temperatures if it is assumed independent of the conduction process, it can be substituted in Equation (1) to give

$$\Theta_i = \frac{T^*_{g,i} - T^*_1}{1 - T^*_1}$$

$$= \frac{\left[\dfrac{\epsilon_0 F_{0-\Delta X,i}(1 - T_1^{*4})}{4\tau\ \Delta X} + N\,\dfrac{(T^*_{g,i-1} - 2T^*_{g,i} + T^*_{g,i+1})}{4\tau(\Delta X)^2} + T_1^{*4}\right]^{1/4} - T^*_1}{1 - T^*_1} \tag{3}$$

where the following definitions have been made and substituted:

$$T^*_A \equiv \frac{T_A}{T_0}; \qquad \tau \equiv \kappa D; \qquad \Delta X \equiv \frac{\Delta x}{D}; \qquad N \equiv \frac{k/D}{\sigma T^3}$$

This equation defines a nondimensional temperature at the midpoint of the gas increment. With the gas increment numbering system $i = 1, 2, \ldots, f$, Equation (3) will only apply for the range of increment numbers $1 < i < f$. Following the same method of derivation as for Equation (3) for gas increments 1 and f gives the boundary equations

$$\Theta_1 = \frac{\left[\dfrac{\epsilon_0 F_{0-\Delta X,1}(1 - T_1^{*4})}{4\tau\,\Delta X} + \dfrac{N(2 - 3T^*_{g,1} + T^*_{g,2})}{4\tau(\Delta X)^2} + T_1^{*4}\right]^{1/4} - T_1^*}{(1 - T_1^*)} \tag{4}$$

$$\Theta_f = \frac{\left[\dfrac{\epsilon_0 F_{0-\Delta X,f}(1 - T_1^{*4})}{4\tau\,\Delta X} + N\dfrac{(T^*_{g,f-1} - 3T^*_{g,f} + 2T_1^*)}{4\tau(\Delta X)^2} + T_1^{*4}\right]^{1/4} - T_1^*}{(1 - T_1^*)} \tag{5}$$

As is shown in the appendix, the exchange factor $F_{0-\Delta X,i}$ can be found for any combination of wall emissivities ϵ_0 and ϵ_1 and any optical thickness τ with a derivation similar to that in (4) if only the exchange factor for black surfaces $\overline{F}_{0-\Delta X,i}$ is known. The relation is

$$\epsilon_0 F_{0-\Delta X,i} = \frac{\overline{F}_{0-\Delta X,i} + 4\overline{F}_{0-1}E_1\tau\,\Delta x}{1 + \overline{F}_{0-1}(E_0 + E_1)} \tag{6}$$

where $E_A = (1 - \epsilon_A)/\epsilon_A$ and $\overline{F}_{0-1}$ is the exchange factor for radiation between black plates through a gray gas.

Examination of Equations (3) to (5) then leads to the conclusion that the gas-temperature distribution $\Theta(X)$ can be found by an iterative solution of Equations (3) to (6) if the conduction-radiation parameter N, the gas optical thickness τ, the plate temperature ratio T_1^*, and the emissivities of the plates are specified. All interchange factors may be obtained from available solutions of pure radiative transfer between black surfaces enclosing a gray gas, some of which are taken from (5) and plotted in figure 1. The equations are seen to involve no integral terms, although $\overline{F}_{0-\Delta X,i}$ was, of course, originally found by some method of solution of a problem involving integral equations.

Heat Transfer Between Surfaces

Once the gas temperature distribution is known from the previous calculations, the heat transfer between the surfaces may be obtained from a heat balance at surface 0, which yields

$$q_{0-1} = -k\left(\frac{dT}{dx}\right)_0 + \epsilon_0\sigma T_0^4 F_{0-1} - \epsilon_1\sigma T_1^4 F_{1-0} - \sum_{g-0} \tag{7}$$

If the surface temperatures are equal, from $\sum_{g-0} \approx 0$ may be found

$$\epsilon_0 F_{0-1} = \epsilon_1 F_{1-0} \tag{8}$$

If it is assumed that F_{AB} is independent of the conduction process, then Equation (8) may be substituted into Equation (7) to obtain

$$\frac{q_{0-1}}{\sigma(T_0^4 - T_1^4)} \approx \frac{-N}{(1 - T_1^{*4})}\left(\frac{dT_g^*}{dX}\right)_0 + \epsilon_0 F_{0-1} \tag{9a}$$

If Equation (9a) is rederived for the other surface, the following is obtained:

$$\frac{q_{0-1}}{\sigma(T_0^4 - T_1^4)} \approx \frac{-N}{(1 - T_1^{*4})}\left(\frac{dT_g^*}{dX}\right)_1 + \epsilon_0 F_{0-1} \tag{9b}$$

The exchange factor $\epsilon_0 F_{0-1}$ can be found from the black surface exchange factor $\overline{F}_{0-1}$ through the relation

$$\epsilon_O F_{O-1} = \frac{\overline{F}_{O-1}}{1 + \overline{F}_{O-1}(E_1 + E_O)} \tag{10}$$

Case of Concentric Cylinders

For infinite concentric gray cylinders enclosing a gray gas, a derivation similar to that for Equations (3) to (5) gives

$$\Theta_i = \frac{\left\{\dfrac{(1 - R_O)\epsilon_C F_{O-\Delta R,i} R_O (1 - T_1^{*4})}{4\tau_c\ \Delta R\ R_i} + \dfrac{N_c(1 - R_O)^2\left[R_{i-\frac{1}{2}}(T^*_{g,i-1} - T^*_{g,i}) - R_{i+\frac{1}{2}}(T^*_{g,i} - T^*_{g,i+1})\right]}{4\tau_c(\Delta R)^2\ R_i} + T_1^{*4}\right\}^{1/4} - T_1^*}{(1 - T_1^*)} \tag{11}$$

$$\Theta_1 = \frac{\left\{\dfrac{(1 - R_O)\epsilon_O F_{O-\Delta R,1} R_O (1 - T_1^{*4})}{4\tau_c\ \Delta R\ R_{i=1}} + \dfrac{N_c(1 - R_O)^2\left[2R_O(1 - T^*_{g,1}) - R_{i=1\frac{1}{2}}(T^*_{g,1} - T^*_{g,2})\right]}{4\tau_c(\Delta R)^2\ R_{i=1}} + T_1^{*4}\right\}^{1/4} - T_1^*}{(1 - T_1^*)} \tag{12}$$

$$\Theta_f = \frac{\left\{\dfrac{(1 - R_O)\epsilon_O F_{O-\Delta R,f} R_O (1 - T_1^{*4})}{4\tau_c\ \Delta R\ R_{i=f}} + \dfrac{N_c(1 - R_O)^2\left[R_{f-\frac{1}{2}}(T^*_{g,f-1} - T^*_{g,f}) - (2T^*_{g,f} - T_1^*)\right]}{4\tau_c(\Delta R)^2\ R_{i=f}} + T_1^{*4}\right\}^{1/4} - T_1^*}{(1 - T_1^*)} \tag{13}$$

where $R_0 \equiv r_0/r_1$ and $R_i = R_0 + [i - (1/2)]\Delta R$. The optical thickness τ_c is now defined by $\tau_c \equiv \kappa(r_1 - r_0)$, and $N_c \equiv \dfrac{k/(r_1 - r_0)}{\sigma T_0^3}$.

Again, as shown in (4), the relation between the gray-plate and black-plate exchange factors is

$$\epsilon_0 F_{0-\Delta R,i} = \frac{\overline{F}_{0-\Delta R,i} + \dfrac{4\overline{F}_{0-1} E_1 \tau_c R_i \ \Delta R}{(1 - R_0)}}{1 + \overline{F}_{0-1}(E_0 + R_0 E_1)} \tag{14}$$

where $\overline{F}_{0-1}$ is the exchange factor for radiation between black concentric cylinders enclosing a gray gas of optical thickness τ_c.

Heat Transfer Between Concentric Cylinders

The heat transfer between the cylindrical surfaces can be derived in a manner similar to that for Equation (9), which gives

$$\frac{q_{0-1}}{\sigma(T_0^4 - T_1^4)} \approx \epsilon_0 F_{0-1} - \frac{N_c(1 - R_0)}{(1 - T_1^{*4})} \left(\frac{dT_g^*}{dR}\right)_{R=R_0} \tag{15a}$$

or, on the basis of the derivative at the outer surface,

$$\frac{q_{0-1}}{\sigma(T_0^4 - T_1^4)} \approx \epsilon_0 F_{0-1} - \frac{N_c(1 - R_0)}{R_0(1 - T_1^{*4})} \left(\frac{dT_g^*}{dR}\right)_{R=1} \tag{15b}$$

The relation between the black and gray surface exchange factors is

$$\epsilon_0 F_{0-1} = \frac{\overline{F}_{0-1}}{1 + \overline{F}_{0-1}(E_0 + R_0 E_1)} \tag{16}$$

as obtained from (4).

DISCUSSION AND RESULTS

The method presented herein depends on the availability of radiant exchange factors for the specific problem being attacked. These factors, as calculated for black surfaces in the infinite flat plate and infinitely long concentric cylinder geometries from (5) and (4), respectively, are shown in Figure 1. From these factors, the exchange factors for any combination of surface emissivities may be calculated by the use of the appropriate Equation (6), (10), (14), or (16).

If the factors for a specific geometry are not available in an exact solution, which itself involves integral equations, the approximation presented by Deissler (6) may be used. It is a modified second-order diffusion approximation, with jump boundary conditions, which yields very good results over a large range of optical thickness. The range of validity is shown in (4) and (5) for the geometries analyzed here. For flat plates, this approximation gives

$$\overline{F}_{0-\Delta X,X} = \frac{4\tau\ \Delta X[3\tau(1 - X) + 2]}{3\tau + 4} \tag{17}$$

$$\overline{F}_{0-1} = \frac{4}{3\tau + 4} \tag{18}$$

and for concentric cylinders

$$\overline{F}_{0-1} = \frac{1}{1/2(R_0 + 1) - \dfrac{3\tau_c R_0 \ln R_0}{4(1 - R_0)} - \dfrac{3(1 - R_0)}{16\tau_c R_0}(R_0^2 - 1)} \tag{19}$$

$$\overline{F}_{0-\Delta R,R} = \frac{4\overline{F}_{0-1}\tau_c R\ \Delta R}{(1 - R_0)}\left[1/2 - \frac{3(1 - R_0)}{16\tau_c} - \frac{3\tau_c \ln R}{4(1 - R_0)}\right] \tag{20}$$

as derived from diffusion solutions presented in (4) and (5) following Deissler.

Deissler's method may also be used to compute exchange factors in other geometries.

Comparison of the results of the present method is made with the exact results of Viskanta and Grosh (1) and (3) and Einstein (2) and (8) and with the approximate solution of Konakov (7). The data from Table I of (1) were corrected to remove some errors in the values of N for certain ranges of T_1^*.

The solution with the present method is easily programmed in general form for the high speed computer by any of a number of standard methods of solution. The curves presented herein were computed by the Newton-Raphson method described for this type problem by Ness (9). The solutions converge extremely rapidly because the matrix of equations has nonzero elements on only three diagonals. Heat-transfer results were computed by fitting a parabola through the three computed temperatures near the wall and by computing the derivative at the wall. Derivatives at each wall were obtained in this manner, and the heat transfer computed by each is shown in Table I.

Because the effect of radiation is exaggerated by this method as shown in the appendix, the predicted temperatures are always higher than the exact solution. It follows that the derivative (dT^*/dX) will always be smaller than the exact solution at surface 0, and larger at surface 1. Examination of Equations (9) and (15) will show that, at surface 1, the predicted heat transfer will therefore always be higher than the exact solution, because the term $\epsilon_0 F_{0-1}$ is also always predicted as too large by this method. At surface 0, since the gradient is too small and $\epsilon_0 F_{0-1}$ too large, the result may be in error by a positive or negative amount. Use of the larger derivative will therefore allow conservative prediction for problems in which minimum heat transfer is the goal since the actual heat transfer will be less than that predicted, while use of the smaller derivative will usually give a more accurate result.

Temperature Distributions

Figures 2(a) and (b) compare temperature distributions in a gas, as calculated by the method of this paper, with those based on solution of the integro-differential equations. Comparison is exact for pure conduction ($N \to \infty$) and pure radiation ($N \to 0$) as required, and the widest deviation occurs at values of the conduction-radiation parameter N for which the temperature profile is intermediate to the limiting curves. As discussed before, the method of solution used in this paper tends to exaggerate the effect of radiative transfer, so that the predicted temperatures are too high.

For decreasing surface emissivity, accuracy is also decreased. Highly reflective walls decrease radiative heat transfer between the surfaces, but increase the value of $F_{A-\Delta X,i}$. This effect is exaggerated proportionately by the approximation used in the present method. This is illustrated in Figure 2(c).

One set of temperature distributions for the concentric cylinder case is shown in Figure 3. The exact solutions were obtained by Einstein (8) for the case of a heat-generating core transferring energy through a gray gas to an outer surface. Temperatures at only four points were evaluated in the gas, and these, plus the derivative at the core-gas interface, were used to define the curve in reference (8). The cylinder was assumed to have a length to diameter ratio of 5, and the temperature profiles presented were at the midpoint of the tube. End effects were assumed negligible. Comparison is made here because no other results are available for combined radiation and conduction in cylindrical geometry.

Heat-Transfer Results

Accuracy of the heat-transfer results was generally quite good for the parallel-plate case, but results became less accurate for cases where the radiative effects strongly affected the temperature distribution. This occurred for cases involving large gas optical thickness, low surface emissivity, and a value of the conduction-radiation parameter N such that radiation and conduction both contributed significantly to total heat transfer.

Comparison of the present results to those obtained by merely summing the contribution of pure radiation ψ_{RAD} and pure conduction ψ_{COND} without regard to their interaction shows that the present method gives about the same accuracy in comparison with the exact solutions if the smaller of the (dT^*/dX) values is used, especially at low surface emissivities. This is true except for the limited regions where the approximation breaks down as noted previously.

The value of the present method is its usefulness for many design cases by the prediction of too large a heat-transfer rate, while the simple additive method predicts too small a rate for parallel plates. The use of these two simple methods gives upper and lower limits for heat transfer.

For concentric cylinders, no solution, except Einstein's (8) for a very few cases, exists in the literature. Pure radiation results from (3) indicate that the neglect of end effects in (8) may have introduced considerable error in the heat-transfer calculations.

Figure 4 shows one set of heat-transfer results for the concentric cylinders. The present results are seen to be physically reasonable, approaching the pure radiation solution at small N_0.

Both the present method and the simple additive method predict too large a heat-transfer rate in this geometry. The present method will always be at least as accurate and generally more accurate, however, because the derivative at either wall will be closer to the exact derivative. The results of Konakov (7) are plotted for comparison. The reason is that the present method predicts a wall derivative between the exact combined solution and the pure conduction solution at surface 0 while the additive method uses the higher pure-conduction derivative.

Other Approximate Solutions

Konakov (7) presents simple equations for the transfer of heat in the geometries studied herein and for the case of concentric spheres. For $\tau \leq 2$, his equations in the nomenclature used herein become

$$\psi = \frac{\tau N(1 - T_1^*)}{(1 - T_1^{*4})} + \frac{1}{E_0 + E_1 + 1} \tag{21}$$

for infinite plates, and

$$\psi_c = \frac{-N_c(1 - T_1^*)(1 - R_0)}{R_0(1 - T_1^{*4})\ln R_0} + \frac{1}{1 + E_0 + R_0 E_1} \tag{22}$$

for infinitely long concentric cylinders. These are from Equations (72) and (92), respectively, of reference (7).

Results calculated by Equation (21) are compared in Table I to exact solutions and are seen to vary widely in many cases from them.

For concentric cylinders, Konakov's equation predicts $\psi = 1$ for pure radiation between black cylinders for all $\tau_c < 2$ and all R_0, which is shown to be in error by up to 150 percent at τ_c near 2 and R_0 near 1 by the results of (4).

For these reasons, Konakov's results should be used with some discretion.

Probstein (10) also takes the approach of adding the independent conduction and radiation solutions without regard to interactions but uses a radiation solution similar to that of Deissler (6) in that an energy jump at the surface-gas interface is considered. Deissler's solution reduces to the radiation solution of (10) for the infinite black plate enclosing a gray-gas case, but is probably more accurate in other geometries because of the inclusion of second-order terms. Deissler (6) also includes the effect of gray walls and nongray gases. The heat-transfer results of (10) are quite similar to those obtained by the addition of the independent radiation and conduction solutions.

CONCLUDING REMARKS

The solutions of certain types of radiant-exchange problems are available in the literature. Where these solutions are present, it should be possible to determine exchange factors, and then to follow a similar analysis to that applied here in order to find solutions to combined radiation and conduction problems in similar geometries.

Results obtained in this manner, especially for the transfer of heat between surfaces, should be better than those attained by the other approximations available. One approximation is simply the addition of the independent results obtained for pure conduction and pure radiation. All interactions are ignored. Although Einstein (2) showed that the heat transfer calculated in this manner differed from the exact solution by less than 10 percent over the range of parameters and for the geometry he studied, it can be shown by the results of Table I (3), that greater deviations occur in other ranges, especially for low surface emissivities.

The method proposed here predicts a gas temperature gradient which is dependent on interaction of conduction and radiation and thus allows fairly accurate prediction of the total heat transfer, with the added bonus of a predicted temperature distribution in the gas which is not available from the other approximations.

The method discussed may also be of value for problems combining radiant transfer with convection, and with convection and conduction. It is not restricted in a general sense to the type problems discussed herein, but could be extended to those involving more complex geometries and less restrictive assumptions on the gas properties. The accuracy of this method remains to be checked in such cases.

As is shown by the results presented herein, the method of this paper predicts temperatures that are somewhat too large and similarly predicts too high a heat transfer in comparison with an exact solution. This allows results that are conservative for many problems to be computed simply.

NOTATION

A	area
D	distance between infinite parallel plates
E_A	emissivity ratio, $(1 - \epsilon_A)/\epsilon_A$
e_b	black body emissive power
$\overline{F}$	exchange factor for black surfaces
F_{AB}	exchange factor, fraction of total energy emitted at surface A that is absorbed at surface B
$F_{A\,\Delta G}$	exchange factor, fraction of total energy emitted at surface A that is absorbed in gas volume increment
k	thermal conductivity of the gas
N, N_c	conduction-radiation parameter, $(k/d)/\sigma T_0^3$ for plates, $[k/(r_1 - r_0)]/\sigma T_0^3$ for cylinders
Q	heat rate
q_0	heat rate per unit area of surface 0
R	nondimensional radius, r/r_1
r	radius
T	temperature, degrees absolute
T^*	nondimensional temperature, T/T_0
X	nondimensional position between plates, $X = x/D$
x	position between plates
ϵ	emissivity of surface
κ	gas absorption coefficient
Θ	nondimensional temperature ratio, $(T^* - T_1^*)/(1 - T_1^*)$

APPENDIX - DERIVATION OF EXCHANGE-FACTOR RELATIONS

Surface Exchange Factors

Writing heat balances at surfaces 0 and 1 of the infinite plate geometry and neglecting conduction give

$$Q_0 = \epsilon_0 e_{b0} A_0 + (1 - \epsilon_0)(Q_0 \overline{F}_{0-0} + Q_1 \overline{F}_{1-0}) \tag{A1}$$

$$Q_1 = (1 - \epsilon_1)(Q_0 \overline{F}_{0-1} + Q_1 \overline{F}_{1-1}) \tag{A2}$$

where plate 1 has been taken at zero temperature, and $\overline{F}_{AB}$ is the fraction of total energy leaving surface A which is incident on surface B with no intermediate reflections from either surface. It is seen that $\overline{F}_{AB}$ is in essence the exchange factor between black surfaces enclosing a gray gas.

The energy absorbed at surface 1 is

$$\epsilon_1(\overline{F}_{0-1} Q_0 + \overline{F}_{1-1} Q_1) = \epsilon_0 e_{b0} A_0 F_{0-1} \tag{A3}$$

where F_{0-1} is the exchange factor for gray surfaces defined in the body of the paper.

We can note further that, for this geometry, and under the assumption of constant gas properties, the following relations hold:

$$\overline{F}_{0-1} + \overline{F}_{0-0} = 1 \tag{A4}$$

$$\overline{F}_{1-0} + \overline{F}_{1-1} = 1 \tag{A5}$$

$$\overline{F}_{1-0} = \overline{F}_{0-1} \tag{A6}$$

Combining (A1) through (A6) to eliminate Q_0, Q_1, $e_{b0}A_0$, $\overline{F}_{1-0}$, $\overline{F}_{1-1}$, and $\overline{F}_{0-0}$, gives

$$\epsilon_0 F_{0-1} = \frac{\overline{F}_{0-1}}{1 + \overline{F}_{0-1}(E_1 + E_0)} \tag{A7}$$

$\sum$ radiation source term for energy conducted to gas elements and radiated away

σ Stefan-Boltzmann constant

τ, τ_c gas optical thickness, κD for plates, $\kappa(r_1 - r_0)$ for cylinders

ψ heat-transfer ratio, $Q_{A-B}/\sigma A_A(T_A^4 - T_B^4)$

Subscripts:

A,B arbitrary surface

c cylindrical case

f gas increment nearest surface 1

G arbitrary gas volume element

g gas

gi emitted from other gas increments, absorbed in increment i

g - 0 emitted from gas increments, absorbed at surface 0

i gas increment index

$\Delta x, \Delta X, \Delta R$ gas volume increment of width Δx, or nondimensional width ΔX or ΔR

0,1 surface 0 or 1

where $E_A = (1 - \epsilon_A)/\epsilon_A$.

Gas Exchange Factors

A radiant heat balance on isothermal gas increment i, again neglecting conduction, gives

$$\epsilon_0 F_{0-\Delta X,i} e_{b0} A_0 = \overline{F}_{0-\Delta X,i} Q_0 + \overline{F}_{1-\Delta X,i} Q_1 \qquad (A8)$$

For black plates, Equation (2) reduces to

$$4\tau \, \Delta X = \overline{F}_{0-\Delta X,i} + \overline{F}_{1-\Delta X,i} \qquad (A9)$$

Combining (A8) and (A9) with (A1) through (A7) to eliminate $e_{b0}A_0$, $\overline{F}_{1-\Delta x,i}$, Q_1, and Q_0 gives

$$\epsilon_0 F_{0-\Delta X,i} = \frac{\overline{F}_{0-\Delta X,i} + 4\overline{F}_{0-1} E_1 \tau \, \Delta X}{1 + \overline{F}_{0-1}(E_0 + E_1)} \qquad (A10)$$

The derivation of similar relations for concentric cylinders is contained in (4).

Overprediction of Gas Temperatures Between Flat Plates

Pure radiation temperature distributions in gray gases between infinite parallel plates are antisymmetrical around the average emissive power of the plates when plotted in terms of emissive power. The value of Θ can therefore be shown to lie only in the range $0.5 \leq \Theta \leq 0.841$ at the point midway between the plates $(X = 0.5)$ for the pure radiation solution. Comparison of any pure radiation solution in this range with the pure conduction solution then indicates that there will always be a greater or equal amount of sinks than sources* in the body of the gas. This occurs because the exact combined solution must lie between or above the separate pure-mode solutions. Any contribution, therefore, to the heat-balance equations by the $\sum$ source term will be more than compensated by decreases in the exchange factor terms by radiation sinks. Neglect of the $\sum$ contribution, therefore, still leaves an overpredicted temperature under the assumptions.

---

*A source for radiation occurs in any increment where the conducted energy input exceeds the conducted energy output; (d^2T/dX^2) is positive. A sink occurs where the converse is true.

REFERENCES

1. Viskanta, R. and Grosh, R. J.: Heat Transfer by Simultaneous Conduction and Radiation in an Absorbing Medium. ASME Jour. Heat Transfer, Feb. 1962.

2. Einstein, T. H.: Radiant Heat Transfer to Absorbing Gases Between Parallel Flat Plates with Flow and Conduction. NASA TR R-154, 1963.

3. Viskanta, R. and Grosh, R. J.: Effect of Surface Emissivity on Heat Transfer by Simultaneous Conduction and Radiation. Int. J. Heat and Mass Transfer 5, Aug. 1962, pp. 729-734.

4. Perlmutter, M. and Howell, J. R.: Radiant Transfer Through a Gray Gas Between Concentric Cylinders Using Monte Carlo. ASME Paper 63-H-13. (To be pub. in ASME Jour. Heat Transfer.)

5. Howell, J. R. and Perlmutter, M.: Monte Carlo Solution of Thermal Transfer Through Radiant Media Between Gray Walls. ASME Paper 62-H-116. (To be pub. in ASME Jour. Heat Transfer.)

6. Deissler, R. G.: Diffusion Approximation for Thermal Radiation in Gases with Jump Boundary Condition. (To be pub. in ASME Jour. Heat Transfer.)

7. Konakov, P. K.: On the Regularities of Composite Heat Transfer. Int. J. Heat and Mass Transfer 2, 1961, pp. 136-149.

8. Einstein, T. H.: Radiant Heat Transfer to Absorbing Gases Enclosed in a Circular Pipe with Conduction, Gas Flow, and Internal Heat Generation. NASA TR R-156, 1963.

9. Ness, A. J.: Solution of Equations of a Thermal Network on a Digital Computer. Solar Energy III, 1, Jan. 1959.

10. Probstein, R. F.: Radiation Slip. Fluid Mech. Lab., M.I.T., Feb. 1963.

TABLE I. - HEAT TRANSFER BETWEEN INFINITE PARALLEL PLATES

$\epsilon_0 = \epsilon_1$	τ	T_1^*	N	$\psi = \frac{q_{0-1}}{\sigma(T_0^4 - T_1^4)}$					
				Exact solutions		Present, using -		$\psi_{RAD} + \psi_{COND}$	(7)
				References					
				(1) and/or (3)	(2)	$\left(\frac{dT^*}{dX}\right)_0$	$\left(\frac{dT^*}{dX}\right)_1$		
1.0	10	0.5	0.011664	0.133	-----	0.129	0.174	0.127	-----
	4	.2	.208	------	0.428	.420	.979	.417	-----
	4	.2	.0208	------	.269	.282	.454	.267	-----
	1	.5	4	2.78	-----	2.60	3.12	2.69	3.13
	1	.5	.4	.863	-----	.805	1.04	.773	1.21
	1	.5	.04	.647	-----	.622	.692	.581	1.02
	1	.5	0	.560	-----	.560	.560	.560	1.00
	1	.1	.4	.990	-----	.825	1.38	.920	1.36
	1	.1	.04	.658	-----	.622	.786	.596	1.04
	1	.1	0	.554	-----	.560	.560	.560	1.00
	.2	.2	.208	------	1.07	1.02	1.16	.997	1.04
	.2	.2	.0208	------	.907	.898	.951	.847	1.00
	.1	.5	40	22.2	-----	22.3	22.3	22.1	3.13
	.1	.5	4	3.07	-----	3.05	3.11	3.03	1.21
	.1	.5	.4	1.15	-----	1.15	1.19	1.13	1.02
	.1	.5	0	.918	-----	.913	.913	.913	1.00
0.5	10	0.5	4	2.26	-----	2.03	4.35	2.23	-----
	10	.5	.4	.328	-----	.488	1.09	.308	-----
	1.0	.5	4	2.56	-----	2.40	2.92	2.40	2.47
	1.0	.5	.4	.487	-----	.577	.835	.480	.546
	1.0	.1	4	3.99	-----	3.59	4.61	3.87	3.93
	1.0	.1	.4	.742	-----	.596	1.18	.627	.693
	.1	.5	4	2.49	-----	2.46	2.52	2.46	.546
	.1	.5	.4	.558	-----	.543	.597	.538	.355
0.1	10	0.5	4	2.25	-----	3.06	5.66	2.04	-----
	10	.5	.4	.317	-----	.884	1.76	.238	-----
	1.0	.5	4	2.39	-----	2.25	2.78	2.18	2.18
	1.0	.5	.4	.419	-----	.412	.686	.263	.265
	1.0	.1	4	3.75	-----	3.44	4.47	3.66	3.65
	1.0	.1	.4	.591	-----	.432	1.03	.417	.412
	.1	.5	4	2.22	-----	2.19	2.25	2.18	.263
	.1	.5	.4	.285	-----	.273	.327	.265	.074

E 2155

FIGURE LEGENDS

Figure 1a. - Exchange factors between black infinite parallel plates.

Figure 1b. - Exchange factors between infinite plate and gas element.

Figure 1c. - Exchange factors between infinitely long concentric black cylinders.

Figure 1d. - Exchange factors between inner black concentric cylinder and gas increment: effect of gas optical thickness. Radius ratio, $R_0 = r_0/r_1 = 0.1$.

Figure 1e. - Exchange factors between inner black concentric cylinder and gas increment: effect of radius ratio. Gas optical thickness, $\tau_c = 2$.

Figure 2a. - Comparison of temperature distributions between flat plates and effect of conduction-radiation parameter. $\tau = 1.0$, $T_1^* = 0.5$, $\epsilon_0 = \epsilon_1 = 1$.

Figure 2b. - Comparison of temperature distributions between flat plates and effect of conduction-radiation parameter. $\tau = 1.0$, $T_1^* = 0.10$, $\epsilon_0 = \epsilon_1 = 1$.

Figure 2c. - Comparison of temperature distributions between flat plates and effect of surface emissivity. $\tau = 1$, $T_1^* = 0.5$, $N = 0.04$.

Figure 3. - Temperature distributions between infinitely long concentric cylinders.

Figure 4. - Heat transfer between infinitely long concentric cylinders enclosing a gray gas: radius ratio, $R_0 = 0.5$; surface emissivities, $\epsilon_0 = \epsilon_1 = 1$; surface temperature ratio $T_1^* = 0.5$; optical thickness, $\tau_c = 2$.

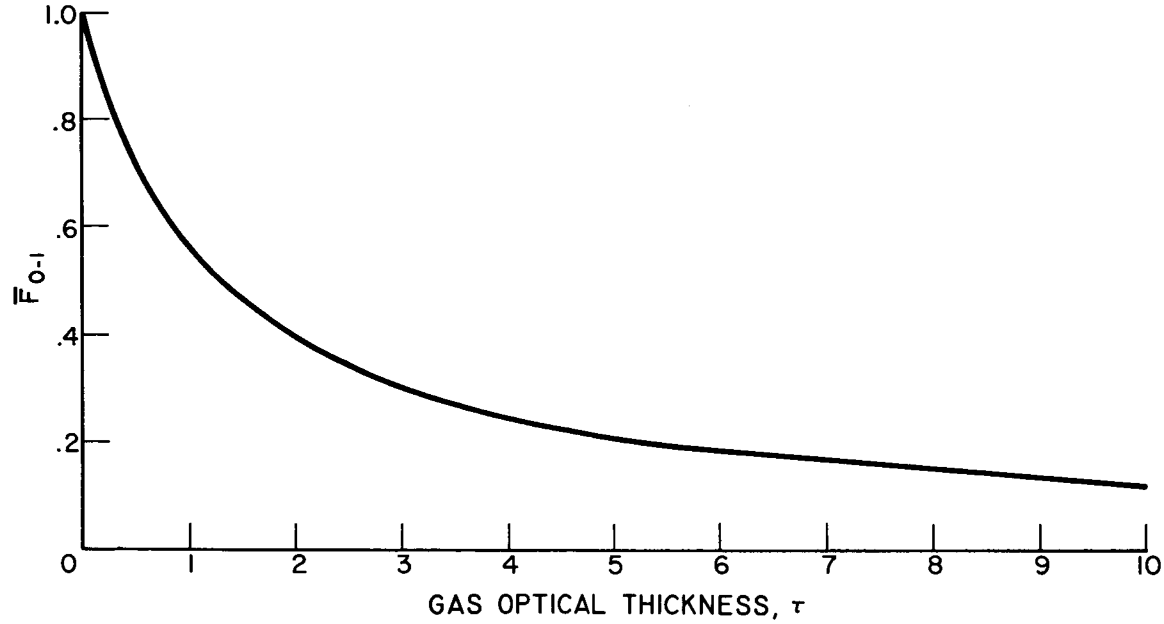

Fig. 1a. Exchange factors between black infinite parallel plates.

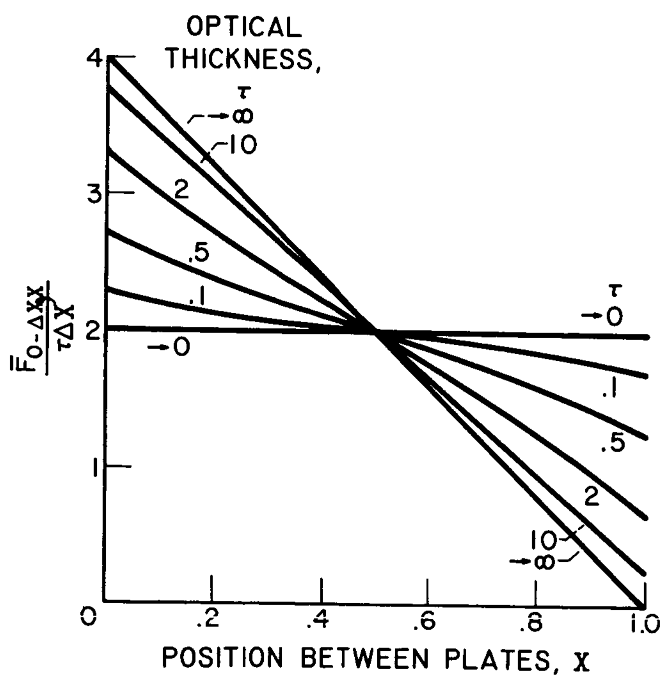

Fig. 1b. Exchange factors between infinite plate and gas element.

E-2155

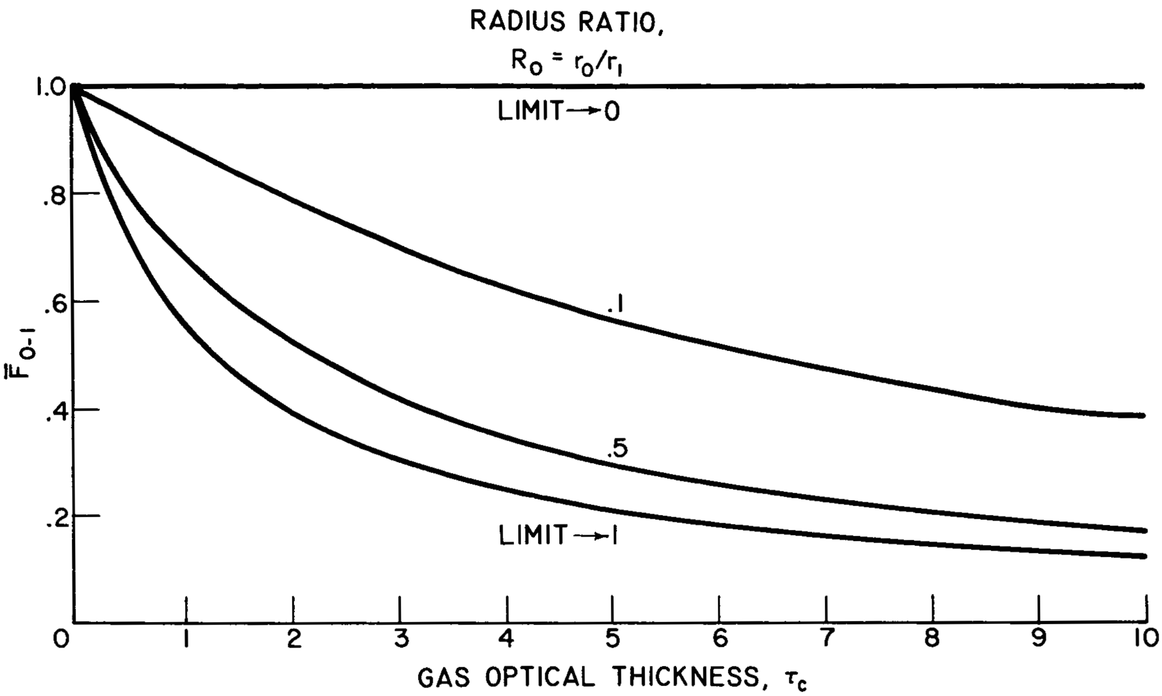

Fig. 1c. Exchange factors between infinitely long concentric black cylinders.

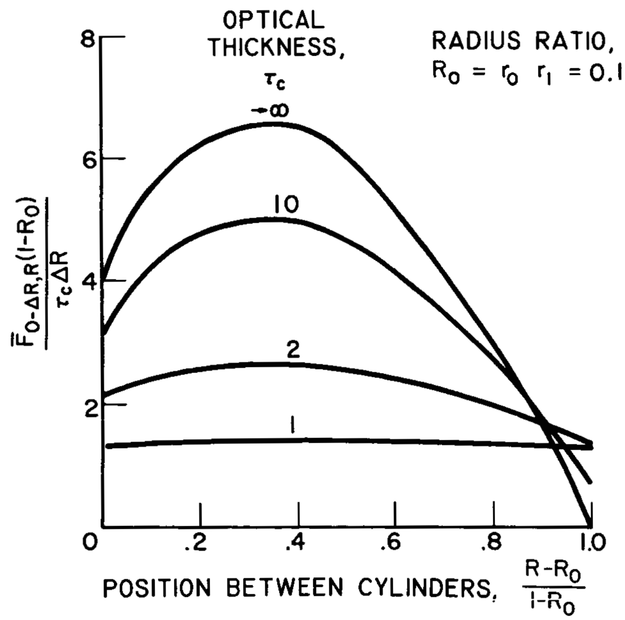

Fig. 1d. Exchange factors between inner black concentric cylinder and gas increment: effect of gas optical thickness. Radius ratio, $R_o = r_o/r_1 = 0.1$.

E-2155

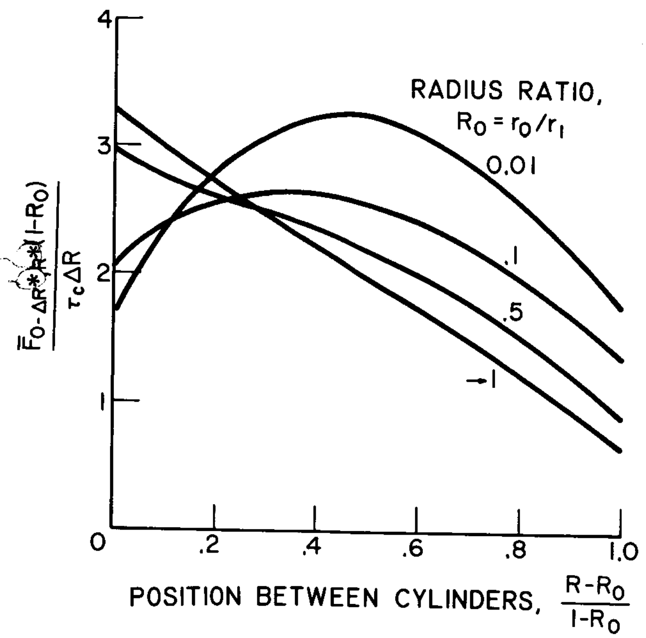

Fig. 1e. Exchange factors between inner black concentric cylinder and gas increment: effect of radius ratio. Gas optical thickness, $\tau_c = 2$.

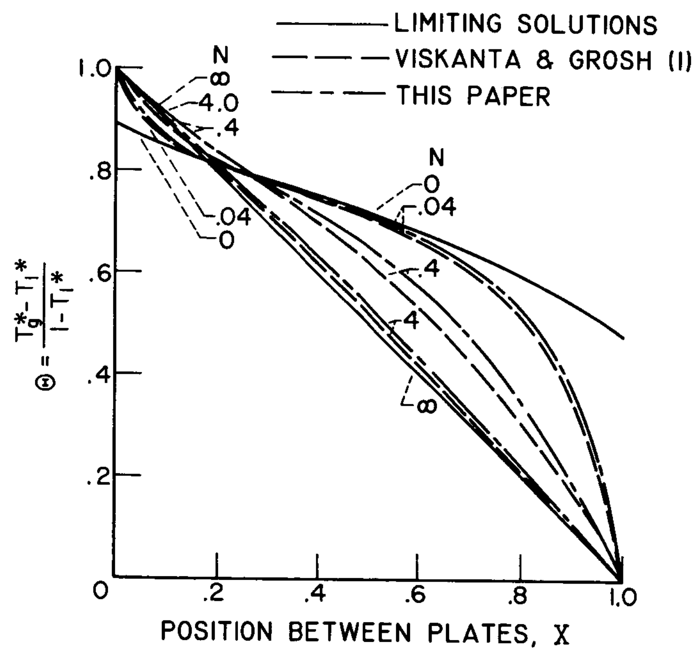

Fig. 2a. Comparison of temperature distributions between flat plates and effect of conduction-radiation parameter. $\tau = 1.0$, $T_1^* = 0.5$, $\epsilon_0 = \epsilon_1 = 1$.

E-2155

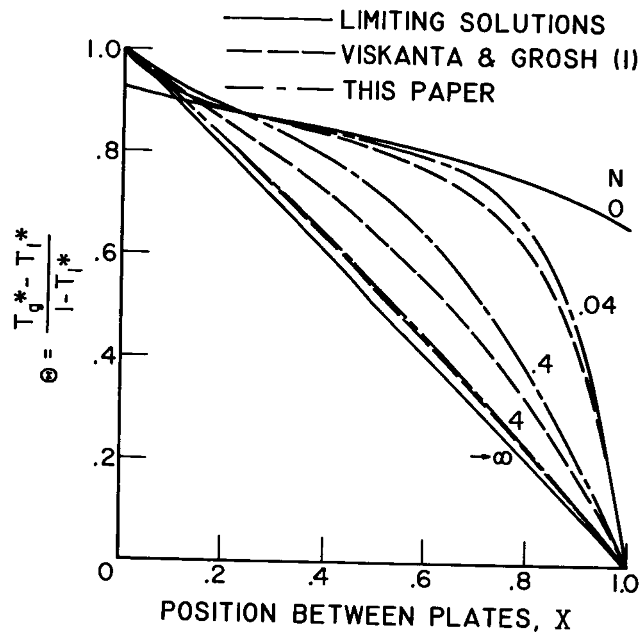

Fig. 2b. Comparison of temperature distributions between flat plates and effect of conduction-radiation parameter. $\tau = 1.0$, $T_1^* = 0.10$, $\epsilon_0 = \epsilon_1 = 1$.

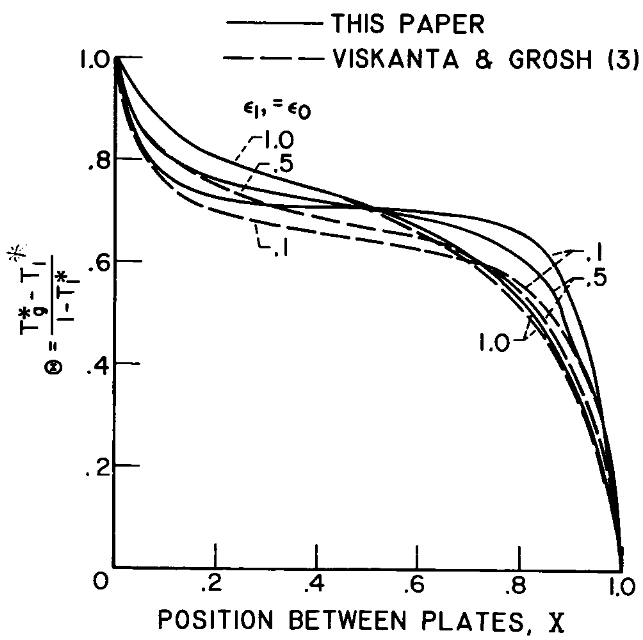

Fig. 2c. Comparison of temperature distributions between flat plates and effect of surface emissivity. $\tau = 1$, $T_1^* = 0.5$, $N = 0.04$.

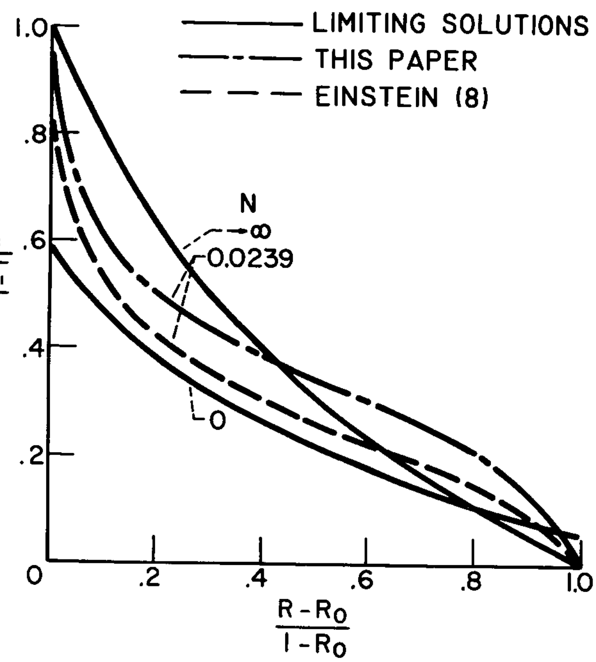

Fig. 3. Temperature distributions between infinitely long concentric cylinders. $T_1^* = 0.576$, $\tau = 1.6$.

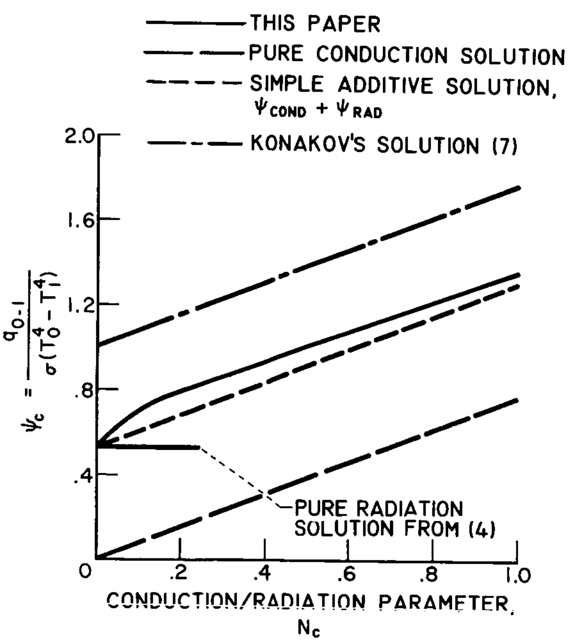

Fig. 4. Heat transfer between infinitely long concentric cylinders enclosing a gray gas: radius ratio, $R_0 = 0.5$; surface emissivities, $\epsilon_0 = \epsilon_1 = 1$; Temperature ratio, $T_1^* = 0.5$; optical thickness, $\tau_c = 2$.